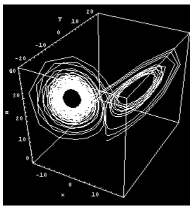
Deterministic Chaos and the Jurassic Park Hypothesis



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Jurassic Park Hypothesis

- The world is too complex to control
- Chaos theory is implicated in the proof by association
- In other words, chaos theory implies that we cannot control complex systems

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Introduction to Deterministic Chaos

- Brief History
- Terminology
- Definition
- Examples
- Control of Chaotic Systems

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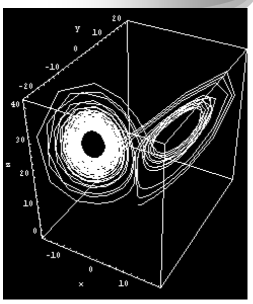
In The Beginning

- January 1889 - Jules Henri Poincare wins King Oscar II's prize for proving stability of the restricted 3-body problem
- November 1889 - Poincare finds an error in the proof. Proves instability instead. (Still wins prize.)

The Modern Era

- 1963 - Edward Lorenz discovers sensitive dependence on initial conditions in now famous "Lorenz equations"
- Early 1970's - First definitions of Deterministic Chaos



Latest Developments

- Control of chaotic systems
- Use of chaos for cryptography
- Studies of "quantum chaos"
- Chaos and complex systems
  - heart (fibrillation)
  - brain (epilepsy)
  - weather

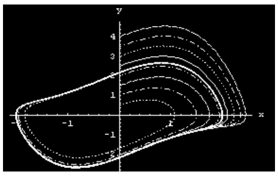
"Definition" of Chaos

There is no unanimously accepted definition of chaos. The most accepted definition by mathematicians is that a system with the following properties is chaotic:

- Sensitive dependence on initial conditions
- Existence of at least one dense orbit
- Set of periodic orbits is dense in phase space

Phase Space and Attractors

- Phase space is a space defined by the system variables, e.g. position and momentum (or velocity).
- An attractor in phase space is a set of points to which nearby orbits are attracted.

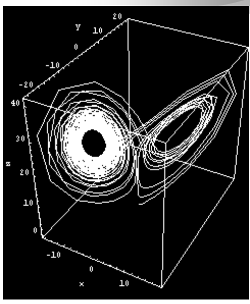


Periodic Orbits

- Periodic orbits are orbits that repeat after a fixed time called the period.
- Periodic orbits appear in bound systems, i.e. systems that do not run away to infinity.
- Periodic orbits are dense in phase space if one exists "next to" any other orbit.

Dense Orbits

An orbit is dense if it passes arbitrarily close to every point in the accessible phase space.



Sensitive Dependence on Initial Conditions

- Two orbits exponentially diverge in phase space on average
- This means that no matter how precise the initial conditions for two orbits of a system are known, the two orbits will differ significantly after a short time

Chaos and Information

- Chaos loses information
  – After some time our knowledge of the system based on our knowledge of the initial conditions is incomplete
- Chaos reveals information
  – After some time our knowledge of the initial state of the system is improved based on the evolution of the system

Bernoulli Shift

$$x_{n+1} = 2x_n \bmod(1)$$

- For a number written in base 2 this shifts bits to the left and then drops the whole number part leaving the fractional part

Results for Bernoulli Map

- Start with 0.11010110 in base two (like your computer)
- 1st iteration: 0.1010110_
- 2nd iteration: 0.010110__
- 3rd iteration: 0.10110___
- 8th iteration: 0.________

What happened?

- We lost our knowledge of the initial condition
- We gained previously unknown information - better knowledge of the initial state of the system

Bernoulli Results Revisted

- 0.11010110
- 1st iteration: 0.1010110n
- 2nd iteration: 0.010110ne
- 3rd iteration: 0.10110new
- etc
- 8th iteration: 0.new_info

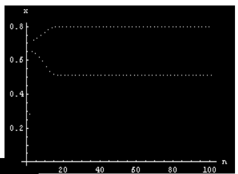
Example - The Logistic Map

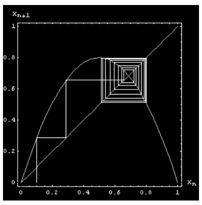
- One of the simplest chaotic systems is the quadratic or logistic map

$$x_{n+1} = r x_n (1 - x_n)$$

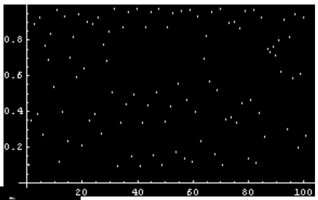
Period 2 Orbit

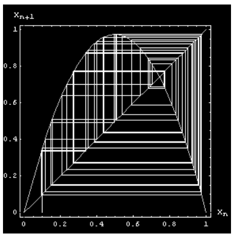
- X as a function of time and a graphic display of the iteration process





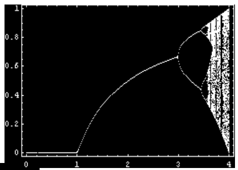
Chaotic Orbit

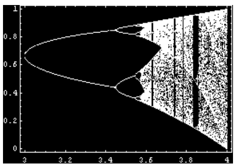




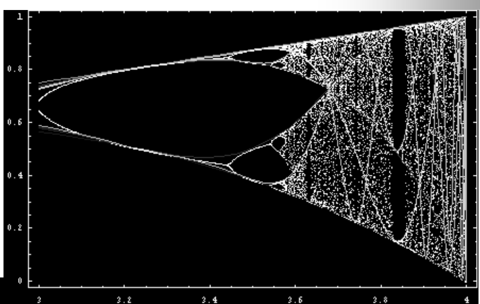
Bifurcation Diagram

- The stability properties for the logistic map change with the value of a parameter, r

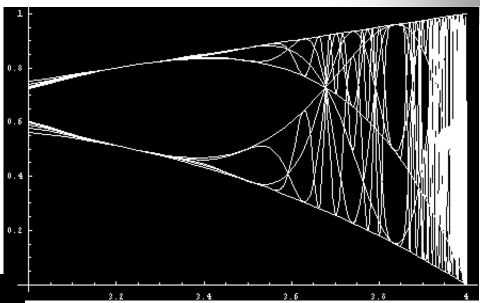




Periodic Orbits for Logistic Map

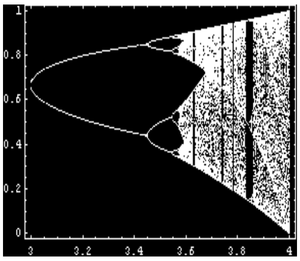


Just the Periodic Orbits



Predictable?

- The orbit visits regions of the x-axis following a regular sequence of steps.
- Fine precision in prediction is difficult
- Coarse prediction is possible

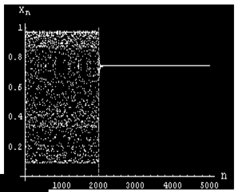


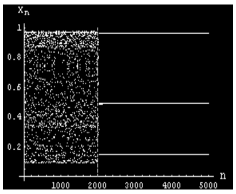
Control Chaos?

- The sensitivity of chaotic systems is useful for controlling such systems
- A small input gives a large effect
- There are many unstable periodic orbits to choose from

Chaotic Control

- By applying small corrections to the system we can control the system





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Conclusions

- Because a system is chaotic, does not mean we have no control over it. We can control a system even with poor predictability in the system
- The methods of chaos theory allow us to simplify some complex systems and possibly control them

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Acknowledgements

- The plots in this talk were made using Mathematica and the chaos package written for Mathematica by Jose Manuel Gutierrez and Andres Iglesias, see Computers in Physics, vol. 12, no. 6, pp. 608-619, Nov/Dec 1998 for more info.

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